UNITED KINGDOM MATHEMATICS TRUST

Open Ended Tasks

These are open ended questions and are not intended for use in a competition format.

It is expected that these will be used in the classroom with groups of pupils and would foster research on the part of the pupils (and maybe the teacher!).

Some guidelines are given but the intention is for the pupils to do some research by whatever means is appropriate, other than simply asking someone else.

Task One

If you have three identical match boxes how many different ways can you stack them face-to-face?

What if you have six match boxes?

Task Two - Part A

i) Write the numbers 1, 2, 3, 4, ...23, 24, 25 in a list.

Square each one of the numbers and write this down next to the original number, as shown below.

Number	Square
1	1
2	4
3	9
4	16
.	.
.	.
24	576
25	625

What do you notice?

Are there any patterns?

Discuss your findings with others on your table.

Now, do the same thing with the numbers 26, 27, 28, 29, ...48, 49, 50. You could use a spreadsheet here.

What patterns do you notice?

Are some the same as with the first set of numbers?

Talk over your findings with others on your table or in your class.

ii) Look at squaring numbers ending in 5.

For instance: $25^2 = 2 \text{ X}3(25) = 6(25) = 625$

$65^2 = 6 \text{ X}7(25) = 42(25) = 4225$

$125^2 = 12 \text{ X } 13 (25) = 156(25) = 15625$

Try some of your own numbers which end in a ‘5’.

Do you notice any patterns?

Talk it over with others on your table.

Describe what is happening. Can you explain it?

Can you explain why?

Task Two- Part B

I know that 214562 is not a square number by just looking at it.

However, I know that 8128 could be a cube number, again by just looking at the number.

How could this be possible?

The last digit gives a clue, but how?

Try these tasks and see what you can find out.

- What are the last digits that a square number cannot end in?
- What about asking the same question for cube numbers?
- What about powers of 4 or 5 or 6.......(you may need to find out what we mean by powers of 4, 5, 6 ....)
- It might be useful to use a spreadsheet here!

Can you find a pattern?

Maybe you should use your knowledge to find out what the last digits of powers of numbers could be?

Test your theory on: 32^2, 105^3, 23^4, 47^5 (check with a calculator)

Can you find the last digits of these without using a calculator? (It is not expect you to have to multiply these out using long multiplication or a spreadsheet)

2013^2, 2013^3, 2013^4, 2013^5, 2013^6......

What about others. Try your own and impress your friends!

Task Three

Look at the expression: $\frac{a}{b} \rightarrow \frac{a+4b}{a+b}$

Start with a = 1 and b = 1 then replace values of a with the top number and values of b with the bottom number.

For example:

$$\frac{1}{1} \rightarrow \frac{1+4X1}{1+1} = \frac{5}{2} \rightarrow \frac{5+4X2}{5+2} = \frac{13}{7} \rightarrow \frac{13+4X7}{13+7} = \frac{41}{20} \rightarrow \frac{41+4X20}{41+20} = \frac{121}{61} \rightarrow .....$$

What do you notice?

Why do you think this sequence approaches a particular number?

What do you think would happen if you replace the '4' with a different number (5, say) and follow a similar flow?

Try with any number and work with your friends to come together with a possible explanation and some conclusions.

Task Four

You have 42 one-pound coins in your purse. They all look the same, but one is counterfeit and is a different weight from the rest which all weigh the same.

You only have a balance available to find the counterfeit coin.

How many times do you need to use the balance to find this coin?

Consider other numbers of one-pound coins which are identical except for one counterfeit coin which is a different weight from the rest.

Are some numbers of coins easier to deal with than others? Which are they?

Why are some numbers harder to deal with than others?

What is the best strategy for finding the counterfeit coin, however many coins you have in your purse?

Guidelines

These are minimal, as we do not wish to point the pupils in any particular direction, as ideas may spontaneously emerge. Though, it is possible that some ideas may emerge that only work in specific cases, there is nothing wrong with this as it can be useful to explain why it only works in specific cases.

We hope that these tasks promote discussion and that they may lead onto other questions and ideas from the pupils (and the teachers!).

It is hoped that some of this research will take place outside the classroom and that the pupils would be enthused and encouraged to follow through on ideas when not confined to the classroom.

When working with the pupils any comments or questions are useful if they start with 'what if..........'

Task One: Use empty matchboxes of identical proportions.

Task Two: Use a spreadsheet but some of the earlier square numbers should be known. The hope is that patterns will help pupils to be able to work out some of the squares without recourse to calculator of spreadsheet.

Task Three: A calculator or a spreadsheet would be useful here.

Square roots here but hope some irrational numbers used and the idea of limits can be explored.

Task Four: A task that like Task One, could be practical (maybe not using real £1 coins though). Emphasis on strategies here and extended to other ideas that require a strategy.